$$= \frac{u_n^2 - u_n^2 - 1}{\sqrt{u_n^2+1}\left(u_n+\sqrt{u_n^2+1}\right)}$$

$$= -\frac{1}{\sqrt{u_n^2+1}\left(u_n+\sqrt{u_n^2+1}\right)} \leqslant 0$$

$\sqrt{u_n^2+1} \geqslant 0$; $u_n+\sqrt{u_n^2+1} \geqslant 0$

Donc $u_{n+1} \leqslant 1$

Conclusion : $\forall n \in \mathbb{N}$; $0 \leqslant u_n \leqslant 1$

b) pour $n=0$; $u_0 = \frac{1}{\sqrt{0+1}} = 1$ (Vraie)

Soit $n \in \mathbb{N}$. supposons que $u_n = \frac{1}{\sqrt{n+1}}$ et montrons que $u_{n+1} = \frac{1}{\sqrt{n+2}}$

On a $u_{n+1} = \frac{u_n}{\sqrt{1+u_n^2}} = \frac{\frac{1}{\sqrt{n+1}}}{\sqrt{1+\frac{1}{n+1}}}$

$$= \frac{\frac{1}{\sqrt{n+1}}}{\sqrt{\frac{n+2}{n+1}}} = \frac{\frac{1}{\sqrt{n+1}}}{\frac{\sqrt{n+2}}{\sqrt{n+1}}}$$

$$= \frac{1}{\sqrt{n+2}}$$